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ABSTRACTS

(Aphabetically ordered by last names)

Commutators and Campanato spaces

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For a function $f \in L^1_{loc}(\mathbb{R}^n)$ and a ball B, let $f_B = \frac{1}{|B|}\int_B f(y)dy$. For a variable growth function $\varphi : \mathbb{R}^n \times (0,\infty) \to (0,\infty)$ and a ball $B = B(x,r)$ we write $\varphi(B) = \varphi(x,r)$.

Definition 1. . *For $p \in [1,\infty)$ and $\varphi : \mathbb{R}^n \times (0,\infty) \to (0,\infty)$, let $\mathcal{L}^{(p,\varphi)}(\mathbb{R}^n)$ be the set of all functions f such that the following functional is finite:*

$$\|f\|_{\mathcal{L}^{(p,\phi)}} = \sup_B \Big(\frac{1}{\varphi(B)}\frac{1}{|B|}\int_B |f(y) - f_B|^p dy\Big)^{1/p},$$

where the supremum is taken over all balls B in $\mathbb{R}^n$.

If $p = 1$ and $\varphi \equiv 1$, then $\mathcal{L}^{(p,\phi)}(\mathbb{R}^n) = \mathrm{BMO}(\mathbb{R}^n)$. If $p = 1$ and $\phi(x,r) = r^\alpha (0 < \alpha \le 1)$, then $\mathcal{L}^{(p,\phi)}(\mathbb{R}^n) = \mathrm{Lip}_\alpha(\mathbb{R}^n)$.

Let T be a Calderon-Zygmund operator. In 1976 Coifman, Rochberg and Weiss proved that, if $b \in \mathrm{BMO}(\mathbb{R}^n)$, then the commutator $[b,T]$ is bounded on $L^p(\mathbb{R}^n)(1 < p < \infty)$. In 1978 Janson proved that, if $b \in \mathrm{Lip}_\alpha(\mathbb{R}^n)$, then $[b,T]$ is bounded from $L^p(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)(0 < \alpha < 1, 1 < p < q < \infty, -1/p + \alpha = -1/q)$. For the fractional integral operator I_α, in 1982 Chanillo proved that, if $b \in \mathrm{BMO}(\mathbb{R}^n)$, then the commutator $[b,I_\alpha]$ is bounded from $L^p(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)(0 < \alpha < n, 1 < p < q < \infty, -n/p + \alpha = -n/q)$.

In this talk, we consider the boundedness and compactness of the commutators $[b,T]$ and $[b,I_\rho]$ on generalized Morrey spaces, where T is a Calderon-Zygmund operator, I_ρ is a generalized fractional integral operator and b is a function in generalized Campanato spaces with variable growth condition. This is a joint work with Eiichi Nakai.

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Maximal estimates for the fractional Schrödinger equation

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In this talk we consider the pointwise convergence problem for the solutions of generalized Schrödinger equations. We establish the associated maximal estimates for a general class of phase functions, which give the pointwise convergence for $f \in H^s(\mathbb{R}^d)$ whenever $s > \frac{d}{2(d+1)}$. Our arguments are based on recent works of Du, Guth, and Li [1] and Du and Zhang [2]. This is a joint work with hyerim Ko.

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Critical case estimates for Fourier multipliers associated with hypersurfaces

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In this talk, I will establish mixed-norm estimates for some Fourier multiplier operators associated with a given hypersurface and emphasis will be placed on the estimate associated with a certain critical index. The main assumption is that a corresponding dispersive estimate holds, and thus the argument works for a reasonably wide class of hypersurfaces. This talk is based on a joint-work with Neal Bez and Sanghyuk Lee.

The soliton potential interaction for NLS

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This talk will address the problem of a soliton interacting with a potential bump in three dimensions. We focus on the asymptotic dynamics of narrow soliton hitting a smooth positive potential with compact support of size 1. In this case by a paper of Jonnson-Froehlich-Gustafson-Sigalabout the motion of soliton in a potential, it is known that if the soliton is very narrow, then for a finite time, its center of mass moves like a classical particle in the potential, and moreover, there is only a small radiation coming out of it. By combining this result for finite time until the soliton is far from the potential , we then further prove that the soliton scatters away, with asymptotically constant speed and a free wave going to infinity. The asymptotic velocity of the soliton will in general be very different from the initial one. Our results crucially depend on the endpoint Strichartz estimates of matrix charge transfer model

which are uniform in an additional variable parameter $\epsilon > 0$. This is a joint work with X. Yao and A. Soffer.

Muti-parameter local Hardy spaces

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Though multi-parameter Hardy space theory has been well developed in the past half century, not much has been studied for a local Hardy space theory in the multi-parameter settings. Such multi-parameter local Hardy spaces can play an important role in studying the boundedness of multi-parameter pseudo-differential operators, multi-parameter singular integrals of non-convolution type, and applications to partial differential equations, etc. In this report, we will give some development about this issue.

Sharp endpoint estimates for Schrödinger groups

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Let L be a non-negative self-adjoint operator acting on $L^2(X)$ where X is a space of homogeneous type with a dimension n. Suppose that the heat operator e^{-tL} satisfies a generalized Gaussian (p_0, p_0')-estimates of order m for some $1 \le p_0 < 2$. In this talk we show sharp endpoint L^p-Sobolev bound for the Schrödinger group e^{itL}, that is for every $p \in (p_0, p_0')$ there exists a constant $C = C(n,p) > 0$ independent of t such that

$$\big\|(I+L)^{-s} e^{itL} f\big\|_p \le C(1+|t|)^s \|f\|_p, \quad t \in \mathbb{R}, \quad s \ge n\big|\frac{1}{2} - \frac{1}{p}\big|.$$

As a consequence, the above estimate holds for all $1 < p < \infty$ when the heat kernel of L satisfies a Gaussian upper bound. This extends the classical results due to Feffermann and Stein, and Miyachi for the Laplacian on the Euclidean spaces $\mathbb{R}^n$.

This is joint work with Peng Chen, Ji Li and Lixin Yan.

A note on discrete Morrey spaces

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In this talk, we discuss several inclusion relations between discrete Morrey spaces. We give a necessary condition for these inclusion relations. We also consider some connections between inclusion relations of discrete Morrey spaces and those of Morrey spaces.

Uniqueness in the Calderón problem and bilinear restriction estimates

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We consider the conductivity equation $\nabla \cdot (\gamma \nabla u) = 0$ on a bounded domain with Lipschitz boundary. The Caldeón problem concerns whether the conductivity γ in the domain can be uniquely determined by the Dirichlet-to-Neumann map. This problem was usually studied under the assumption that conductivity has bounded gradient. Recently, Haberman showed uniqueness remains valid even for conductivities with unbounded gradients. His result relies on L^2 restriction estimate for hypersurfaces which is known as the Stein-Tomas restriction theorem. In this talk we show that the regularity assumption on conductivity can be relaxed by making use of the sharp bilinear restriction estimates. This talk is based on a joint work with Yehyun Kwon and Sanghyuk Lee.

The boundedness of the bilinear fractional integral operator on Morrey spaces by average technique

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We investigate the boundedness of the bilinear fractional integral operator introduced by Grafakos. When the local integrability index s falls 1 with weights and t exceeds 1, He and Yan obtained some results on this operator was worked on Morrey spaces earlier. Later, we considered the case $t = 1$. This paper handles the remaining case $0 < t < 1$.

On the Korteweg-de Vries limit for the Fermi-Pasta-Ulam system

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The Fermi-Pasta-Ulam (FPU) system is a simple nonlinear dynamical lattice model describing a one-dimensional chain of vibrating strings with nearest neighbor interactions. This model was introduced by Fermi, Pasta and Ulam in 1955. It was anticipated at that time that chaotic nonlinear interactions would lead thermalization. Surprisingly however, numerical simulations showed the opposite behavior – it exhibited quasi-periodic motions. This phenomena is known as the FPU paradox. This puzzle has been solved by Zabusky and Kruskal by discovering a formal convergence to the Korteweg-de Vries equation, and later the convergence has been rigorously justified. We revisit this convergence problem, and show how to put it into the dispersive PDE framework. This talk is based on joint work with Chulkwang Kwak and Changhun Yang.

Limit theorems and wrapping transformations in bi-free probability theory

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In classical probability, Lvy and Khintchine demonstrated that the limit law associated with any triangular array of infinitesimal random variables is infinitely divisible. In this talk, we shall manifest the analogous results for distributions on the plane and bi-torus in the framework of bi-free probability theory. Like the classical situation, bi-freely additive and multiplicative infinitely divisible distributions, and solely these distributions serve as the limiting distributions of a triangular array of infinitesimal random variables. The bi-free harmonic analysis developed by ourselves performs an essential role in the study of bi-free limit theorems. These limit theorem consequences also establish tight bonds between classical and bi-free probability theories. If time permits, some other relevant topics will be discussed.

Sufficient and necessary conditions for Holder's inequality in weighted Orlicz spaces

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Weighted Orlicz spaces are generalizations of Orlicz spaces. Recently, Ifronika *et al.* obtained sufficient and necessary conditions for generalized Hölder's inequality in Orlicz Spaces. Motivated by this result, we would like to disscus the sufficient and necessary conditions for generalized Hölder's inequality in Weighted Orlicz spaces which are obtained through estimates for characteristic functions in $\mathbb{R}^n$.

Boundedness of bilinear pseudo-differential operators of $S_{0,0}$-type

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In this talk, we extend the known result, the $L^2 \times L^2 \to h^1$-boundedness of bilinear pseudo-differential operators with symbols belonging to the Hörmander class $BS^{-n/2}_{0,0}$, in two ways. Firstly, we improve the target space h^1 to a wider space. Secondly, we generalize the the symbol class $BS^{-n/2}_{0,0}$ by replacing the weight function $(1 + |\xi_1| + |\xi_2|)^{-n/2}$ appearing in $BS^{-n/2}_{0,0}$ by general functions. This talk is based on a joint research with Prof. Miyachi (Tokyo Woman's Christian University) and Prof. Tomita (Osaka University).

Singular integrals for Jacobi analysis

Takeshi Kawazoe
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We shall consider the singular integrals of convolution type on the Jacobi hypergroup $(\mathbf{R}_+, \Delta_{\alpha,\beta}, *)$. In the Euclidean case, if a function g satisfies (1) the Fourier transform is bounded, (2) the smoothness condition, then the operator $T_g(f) = f * g$ is bounded from L^p to itself for $1 < p < \infty$. In this talk we shall define a Calderón-Zygmund class $\mathrm{CZ}(\Delta_{\alpha,\beta})$ on the Jacobi hyper group for which, if a function g on $\mathbf{R}_+$ belongs to $\mathrm{CZ}(\Delta_{\alpha,\beta})$, then the convolution operator $*g$ is bounded from $L^p(\Delta_{\alpha,\beta})$ to itself for $1 < p \leq 2$. Actually, we shall obtain a relation between the L^p norms of g and the Abel transform $\mathcal{A}g$ and a transference principle between the L^p operator norms of $*g$ and $\circledast\phi$, where $\phi = e^{(\frac{2}{p}-1)\rho x}\mathcal{A}(g)$ and $\circledast$ the Euclidean convolution. A partial result on the case that $\alpha - \beta \geq 1$ or $\beta + \frac{1}{2} \geq 1$ was announced last year.

On the radius of spatial analyticity for defocusing nonlinear Schrödinger equaitons

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Consider the defocusing nonlinear Schrödinger equation,

$$iu_t + \Delta u = |u|^{p-1}u,$$

where $p > 1$ is an odd integer. Our attention is focused on the situation where we consider a real-analytic initial data with a uniform radius of analyticity, so there is a holomorphic extension to a complex strip. It is now natural to ask whether this property may be continued analytically to a complex strip for solutions at later times t, but with a possibly smaller and shrinking radius of analyticity. In this talk, we will show that the uniform radius of spatial analyticity of solutions at later times t cannot decay faster than $1/|t|$ as $|t| \to \infty$. This extends the previously known result for the cubic case $p = 3$ to the cases where p is any odd integer greater than 3. This is joint work with Jaeseop Ahn and Ihyeok Seo.

Strichartz estimates for the wave equation in Wiener amalgam spaces

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Recently, Strichartz estimates in Wiener amalgam spaces have been studied for the Schrödinger equation. These spaces, unlike the Lebesgue spaces, control the local regularity of a function and its decay at infinity separately. This separability makes it possible to perform a finer analysis of the local

and global behavior of the solution. But, in the case of the wave equation, it seems difficult to proceed in the previous way used for the Schrödinger case. In this talk, we present the Strichartz estimates in Wiener amalgam spaces for the wave equation. Our approach is based on the integral kernel estimates for the wave propagator which are obtained by making use of Bessel functions, and works for the Schrödinger equation as well. This is joint work with Youngwoo Koh and Ihyeok Seo.

On a covergent problem of multiple Fourier series of radial functions.

Shigehiko Kuratsubo
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For a function ϕ with compact support on R, let $F_\phi(x) := \phi(|x|)$ and $f_\phi(x)* =$ the periodization of $F_\phi(x)$.Our aim is to give the necessary and sufficient conditions for Fourier series of $f_\phi(x)$ to converge everywhere.

Sharp estimates for Laplacian resolvent outside of the uniform boundedness range

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In this talk we are concerned with resolvent estimates for the Laplacian Δ in Euclidean spaces. Uniform resolvent estimates for Δ were shown by Kenig, Ruiz and Sogge (1987, Duke Math.) who established rather a complete description of the Lebesgue spaces allowing such estimates. However, the problem of obtaining sharp $L^p - L^q$ bounds depending on the spectral parameter z has not been considered in a general framework which admits all possible p, q. We present a complete picture of sharp $L^p - L^q$ resolvent estimates, which may depend on z. This is a joint work with Sanghyuk Lee.

A trilinear approach to square function and local smoothing estimates for the wave operator

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In this talk we present the estimates for Mockenhaupts square function in R^3 and for Sogges local smoothing in R^{2+1} spacetime. We introduce the trilinear approach of S. Lee and A. Vargas for the cone multiplier and show how to combine their approach with the ℓ^2 decoupling theorem and multilinear restriction theorem.

Local well-posedness for the inhomogeneous nonlinear Schrödinger equation in the critical case

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Consider the inhomogeneous nonlinear Schrödinger equation (INLS),

$$i\partial_t u + \Delta u = \lambda |x|^{-\alpha} |u|^{\beta} u,$$

where $\alpha, \beta > 0$ and $\lambda = \pm 1$. The well-posedness of this equation with initial data in Sobolev spaces H^s has been intensively studied in recent years and is well-understood for the non-critical case. But it remains open for the critical case, $s = s_c$, with the critical Sobolev index $s_c = d/2 - (2-\alpha)/\beta$. In this talk, we handle this critical case by approaching to the matter in a weighted setting. Our approach is based on Strichartz estimates in weighted L^p spaces and also cover the non-critical case as well. Therefore it seems to be more suitable to perform an analysis of the solution for the INLS model. This is joint work with Jungkwon Kim and Ihyeok Seo.

Hardy space theory, commutators and applications in the multi-parameter flag setting

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Multi-parameter flag setting was introduced by Müller, Ricci and Stein when they studied the L^p boundedness of Marcinkiewicz multipliers $m(L, iT)$ on the Heisenberg groups, where L is the sub-Laplacian and T is the central invariant vector field, with m being a multiplier of Marcinkiewicz-type. This flag structure is not explicit, but only implicit in the sense that one can not formulate its dilation. Later, Nagel, Ricci, Stein and Wainger established the L^p theory of singular integrals with flag kernels in a more general setting of homogeneous groups and proved that such singular integrals form an algebra. We established the Hardy space theory by providing the full characterisations via Littlewood–Paley theory, non-tangential and radial maximal functions, flag Riesz transforms as well as atomic decompositions. Based on this, we further studied the boundedness of flag commutators, which gives the flag div-curl lemmas, and has potential links to the Hankel operators in the complex setting. This talk is based on recent two joint works with Xuan Thinh Duong, Yongsheng Han, Mingyi Lee, Yumeng Ou, Jill Pipher and Brett Wick.

Maximal function characterizations of Hardy spaces on some open sets

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The theory of Hardy spaces over $\mathbb{R}^n$ was originated by Fefferman and Stein [Acta Math., 1972], which was generalized several years ago to the case of proper subsets of $\mathbb{R}^n$. The pioneering work of generalization was done by Jonsson, Sjögren, and Wallin [Studia Math., 1984] for the case of suitable closed subsets and by Miyachi [Studia Math., 1990] for the case of open subsets. In this article we study Hardy spaces over certain open subsets $\Omega \subset \mathbb{R}^n$. We first define the Hardy space on Ω by means of atoms, and then give different maximal function characterizations.

Global existence for semilinear damped wave equations in relation with the Strauss conjecture

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We study the global existence of solutions to semilinear wave equations with power-type nonlinearity and general lower order terms on n dimensional nontrapping asymptotically Euclidean manifolds, when $n = 3, 4$ as well as two dimensional Euclidean space. In addition, we prove almost global existence with sharp lower bound of the lifespan for the four dimensional critical problem.

Multiple Fourier series and lattice point problems

Eiichi Nakai
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For the multiple Fourier series of radial functions, we investigate the behavior of the spherical partial sum. We show the Gibbs-Wilbraham phenomenon, the Pinsky phenomenon and the third phenomenon for the multiple Fourier series. The third phenomenon is closely related to the lattice point problems, which is a classical theme of the analytic number theory. This is a joint work with Professor Shigehiko Kuratsubo.

The pointwise convergence of the one-dimensional Schrödinger equation

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Carleson's problem for the Schrödinger propagator asks the behavior of one (quantum) particle when $t \to 0$. In this talk, we consider the same problem but for infinitely many particles without any interaction between them. This talk is based on the collaborative work with Professors Neal Bez and Sanghyuk Lee.

Mixed Morrey space and its application

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In this talk, we introduce mixed Morrey spaces and present some basic properties. These properties extend the classical ones. On these spaces, we investigate the boundedness of the iterated maximal operator, which is obtained by repeatedly acting the one-dimentional maximal operator. Furthermore, as a corollary, we obtain the boundedness of the iterated maximal operator on classical Morrey spaces.

T1 theorem for homogeneous Triebel-Lizorkin spaces

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A T1 theorem is an L^2 boundedness criterion of singular integral operators, given by G. David and J. L. Journe in 1984. In this talk, we expand T1 theorem for homogeneous Triebel-Lizorkin spases by using theory of $\mathcal{Q}$ spaces, which are spaces generalize BMO.

Fourier multiplier theorems for a vector-valued function space with sharp regularity condition

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In this talk we will study a vector-valued version of Hormander's multiplier theorem. Our result improves the result of Triebel and extends to the case $p = \infty$ in the scale of Triebel-Lizorkin space. We will also construct some counter examples for the sharpness of our results.

Compactness for commutators of the Calderon-Zygmund operators

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Let T be the θ-type Calderón-Zgymund operator with Dini condition. In this talk, we will show that for $b \in \mathrm{CMO}(\mathbb{R}^n)$, the commutator generated by T with b and the corresponding maximal commutator, are both compact operators on $L^p(\omega)$ spaces, where ω be the Muchenhoupt A_p weight function and $1 < p < \infty$.

Local smoothing estimates for the Klein-Gordon equation with potential

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In this talk, we discuss the following local smoothing estimates for the Klein-Gordon equation perturbed with potential V:

$$\sup_{R>0} \frac{1}{R} \int_{|x|<R} \int_{-\infty}^{\infty} \left| |\nabla|^{1/2} e^{it\sqrt{-\Delta+V+1}} f \right|^2 dt dx \lesssim \|f\|_{H^{1/2}}^2,$$

which have been obtained for potentials which decay faster than the inverse square potential $|x|^{-2}$. But recent studies for the perturbed dispersive equations have intensively aimed to get closely to the inverse square potential. Motivated by this, we improve the previously known results to the potential $|x|^{-2}$. This is joint work with Hyeongjin Lee and Ihyeok Seo.

Two weight T1 theory in higher dimensions

Chun-Yen Shen

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In this talk we present our recent advances showing the two weight T1 theory for Riesz transforms when one measure is supported on a curve. We will also discuss some of the difficulties for general cases.

Nelson's hypercontractity inequality by a diffusion flow method

Shobu Shiraki

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One of the famous classical inequalities regarding the Ornstein-Uhlenbeck semigroup in quantum physics, Nelson's hypercontractivity inequality, has been studied from many different perspectives. We will give a new approach based on a diffusion flow method and also consider the more general setting of Markov semigroups. This is based on joint work with Yosuke Aoki, Jonathan Bennett, Neal Bez, Shuji Machihara and Kosuke Matsuura.

Extremizability of Fourier restriction to the paraboloid

Betsy Stovall

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We will show that for almost all valid Lebesgue space bounds for the restriction operator associated to the paraboloid, there exist nonzero functions whose restrictions have the maximum possible norm. As time permits, we will also discuss related questions for other Fourier restriction/extension operators.

Block decomposition and weighted Hausdorff content

Hitoshi Tanaka

Tsukuba University of Technology, Hitoshi Tanaka

Block decomposition of L^p spaces with weighted Hausdorff content is established for $0 < p \leq 1$ and the Fefferman-Stein type inequalities are shown for fractional integral operator and some variants of maximal operators.

Asymptotic properties of steady solutions to the 2D Navier-Stokes equations with finite generalized Dirichlet energy

Yutaka Terasawa

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We consider the stationary Navier-Stokes equations in the exterior 2D domain outside of a large circle. The solution v is handled in the class with $\nabla v \in L^q$ for $q \geq 2$. Since we deal with the case $q \geq 2$, our condition for the spatial decay of ∇v is weaker than the condition of the finite Dirichlet integral, i.e., the case $q = 2$, where a number of results such as asymptotic behavior of solutions have been observed. For the stationary problem we shall show that $\omega(x) = o(|x|^{-(1/q+1/q^2)})$ and $\nabla v(x) = o(|x|^{-2/q-1/q^2+1/2})$ as $|x| \to \infty$, where $\omega \equiv \mathrm{rot} v$. This talk is based on a joint work with Professor Hideo Kozono (Waseda University) and Professor Yuta Wakasugi (Ehime University).

Fefferman's Inequality and Unique Continuation Property of Elliptic Partial Differential Equations

Nicky Kurnia Tumalun

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In this talk we prove a Fefferman's inequality for potentials belonging to a generalized Morrey space $L^{p,\varphi}$ and a Stummel class $\tilde{S}_{\alpha,p}$. Our result extends the previous Fefferman's inequality that was obtained in [2, 3] for the case of Morrey spaces, and that in [4] for the case of Stummel classes, which was restated recently in [1]. Using this inequality, we prove a strong unique continuation property of a second order elliptic partial differential equation that generalizes the result in [1] and [4].

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The sparse form bounds for Fourier integral operators

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Given $a \in S^m_{\rho,\delta}$ and $\Phi \in C^\infty(\mathbb{R}^n \times (\mathbb{R}^n \setminus \{0\}))$, Hörmander [3] defined Fourier integral operator (FIO for short) $T_{a,\Phi}$ is defined as

$$T_{a,\Phi}f(x) = \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n} e^{i\Phi(x,\xi)}a(x,\xi)\hat{f}(\xi)d\xi$$

for $f \in \mathcal{S}$. Aim of my study is to establish the sparse form bounds for $T_{a,\Phi}$ with $a \in S^m_{1,0}$:

$$|\langle T_{a,\Phi}f, g\rangle| \lesssim \Lambda_{\mathcal{S},r,s'}(f,g) := \sum_{Q\in\mathcal{S}} |Q|\langle f\rangle_{r,Q}\langle g\rangle_{s,Q}$$

where $\langle f\rangle_{p,Q} := |Q|^{-\frac{1}{p}}||f||_{L^p(Q)}$ and $\mathcal{S}$ denote the sparse collection. In the case $\Phi(x,\xi) = x\xi$, $T_{a,\Phi}$ is a pseudodifferential operator. Beltran and Cladek [2] proved the spares form bounds for pseudodifferential operators with symbols in $S^m_{\rho,\rho}$ with $0 \le \rho < 1$ and suitable m. It is natural to ask the same problem for FIOs instead of pseudodifferential operators. Furthermore, we study the weighted L^p-boundedness with Muckenhoupt weight class of FIOs. Bernicot, Frey and Petermichl [1] showed

$$\Lambda_{\mathcal{S},r,s'}(f,g) \lesssim ([\omega]_{A_{p/r}}[\omega]_{RH_{(s/p)'}})^{\alpha}||f||_{L^p(\omega)}||g||_{L^{p'}(\omega^{1-p'})}$$

where $\alpha = \max(\frac{1}{p-r}, \frac{s-1}{s-p})$, $[\omega]_{A_q} = \sup_Q\langle\omega\rangle_{1,Q}\langle\omega^{1-q'}\rangle^{q-1}_{1,Q}$ and $[\omega]_{RH_q} = \sup_Q\langle\omega\rangle^{-1}_{1,Q}\langle\omega\rangle_{q,Q}$ for any $1 < q < \infty$. This estimate and sparse form bounds for FIOs give us the weighted L^p-boundedness with Muckenhoupt weight class of FIOs.

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Continuum limit of discrete Schrdinger equations

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In this talk, we consider discrete nonlinear Schrdinger equations(DNLS) on the lattice whose linear part is determined by the discrete Laplacian which accounts for nearest neighbor interactions. We show that in the continuum limit, solutions to DNLS converge strongly in L2 to those to the corresponding continuum equations. Our proof is based on a suitable adjustment of dispersive PDE techniques to a discrete setting. Notably, we employ uniform

Strichartz estimates for DNLS, which quantitatively measure dispersive phenomena on the lattice. This is joint work with Younghun Hong, Chulkwang Kwak and Shohei Nakamura.

Decay estimates for higher order elliptic operators in the presence of zero resonance or zero eigenvalue

Xiaohua Yao

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In this talk, I will report some progress on the decay estimates and related problems for the higher-order elliptic operators $H = (-\Delta)^m + V(x)$ in $\mathbf{R}^n$ for $n > 2m$ and $m \geq 2$. For certain decay potentials $V(x)$, we mainly talk about Kato-Jensen type decay estimates for higher order groupr e^{-itH} in the presence of zero resonance or zero eigenvalue. Among them, the endpoint Strichartz estimate and L^p-decay estimates can also be mentioned. Our methods heavily depend on the asymptotic expansions of resolvent $R_V(z)$ near zero threshold in the presence of zero resonance or zero eigenvalue. This is some joint works with Hongliang Feng, Avy Soffer and Zhao Wu.

A decoupling interpretation of efficient congruencing in 3 dimensions

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In 2014 Wooley gave a proof of the cubic case of Vinogradov mean value theorem. The proof was subsequently simplified by Heath-Brown. In 2015, Bourgain, Demeter and Guth gave a proof of the general case of Vinogradov mean value theorem, by establishing decoupling inequalities for the moment curve in all dimensions. In this talk, we will discuss a decoupling interpretation of efficient congruencing in 3 dimensions. In particular, inspired by the work of Heath-Brown, we will give a new proof of an $\ell^4 L^{12}$ decoupling inequality for the moment curve in 3 dimensions. This is joint work with Shaoming Guo and Zane Li, and is based in turn on earlier work of Zane Li in 2 dimensions.